TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. t^n; n = positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11. $t^n e^{at}$, n = positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$

Laplace Transform Theorem

$L\{f^{(n)}(t)\} = s^nL\{f(t)\} - s^{n-1}f(0) - sf^{n-2}(0) - f^{(n-1)}(0)$

Every term, s--, and f's derive++, to where s = 0

Example: $y'' - y' - 2y = 0$

$s^2L\{y\} - s^1y(0) - s^0y'(0) - [sL\{y\} - y(0)] - 2L\{y\} = 0$

Example: $y'''' - y = 0$. $y(0) = 0$, $y'(0) = 1$, $y''(0) = y'''(0)=0$

$F = S^4F - s^3(0) - s^2(1) - s(0) - 0 \rightarrow s^4F - s^2 - F = 0$

$F = s^2 / (s^4 - 1) = .5 / (s^2+1) + .5 / (s^2 - 1)$

$y = ½ \sin t + ½ \sinh t$

Power Series Solutions – Assume $y = \Sigma A_n(x - x_0)^n$

Example: $y'' - xy = 0$

$y = \Sigma a_nx^n \qquad y' = \Sigma (n+1)a_{n+1}x^n \qquad y'' = \Sigma (n+1)(n+2)a_{n+2}x^n$

$y'' - xy = \Sigma (n+1)(n+2)a_{n+2}x^n - x^* \Sigma a_nx^n = \Sigma (n+1)(n+2)a_{n+2}x^n - \Sigma a_nx^{n+1} =$

$\Sigma(n = 0)(n+1)(n+2)a_{n+2}x^n - \Sigma(n = 1)a_{n-1}x^n =$

$2a_2 + \Sigma(n = 1)(n+1)(n+2)a_{n+2}x^n - \Sigma(n = 1)a_{n-1}x^n = 0$

Hence, $2a_2 = 0$, $(n+1)(n+2)a_{n+2} = a_{n-1}$ – recurrence relation, steps of 3

Radius of convergence

$y'' + py' + qy = 0$. If p has a r.o.c of r_1, q has a r.o.c of r_2, then the r.o.c of y is at least the smaller of r_1 and r_2. See power series for r.o.c's of functions

Example: $y'' + \ln(1+2)y' + \cos y = 0$. $\ln(1+x)$ has a r.o.c of 1 (power series)

Cos x has a r.o.c of infinite (power series). Y has a r.o.c of at least 1

For complex roots, graph them on real/imaginary axis, find distance using pythigori

Modeling Problems

Interest – $dS/dt = rS$ – fixed rate interest

Solution = S_0e^{rt}

Water in a tank – dQ/dt = rate in – rate out

Salt enters at 1/4lb/gal in r gal/min. salt leaves at r*Q/100. $dQ/dt = r/4 - dQ/100$

Escape Velocity = $v(dv/dx) = -(gR^2)/(R+x)^2$ – separable

$V = +/- \text{Rad}(v_0^2 - 2gR + 2gR^2/(R+x))$