Tests for Convergence/Divergence

1. **Divergence Test**
   a. If the limit of a sequence/series doesn't not approach 0 or doesn't exist, it DIVERGES
   b. Note: Just because the limit is 0 does NOT mean the series converges
2. **Continuous Test**
   a. $a_n = f(n)$. $\lim a_n = \lim f(x)$
3. **Comparison Test**
   a. If $a_n < b_n$, and b_n converges, then a_n must also converge
   b. If an < bn and bn is divergent, then an diverges
   c. If $\lim (a_n/b_n) = c$, then either both series converge or both diverge
4. **Monotonic Test**
   a. If a sequence is monotoic/bounded, it converges
5. **Integral Test**
   a. $a_n = f(x)$. Σa_n converges if $\int_1^\infty f(x)$ converges
   b. Doesn't necessarily have to start at 1. If it converges for any number a, the series converges.
6. **P Series Test**
   a. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$, divergent if $p \le 1$
7. **Ratio Test**
   a. $|a_{n+1}| / |a_n| < 1$, Converges
8. **Root Test**
   a. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then $\sum_{n=1}^{\infty} a_n$ is abs convergent
   Otherwise, if $L > 1$, the series diverges.
9. **Alternating Series Test**
   a. If the series is decreasing and the limit approaches 0, then the series converges
   b. ALWAYS check to make sure that the series is decreasing. DO NOT ASSUME
   c. Note: 1st few terms never influence convergence

Estimating Sums

1. **Alternating Series**
   a. $| R_n | = | s - s_n | < b_{n+1}$
   b. $| R_n | = | s - s_n | > -b_{n+1}$
   c. Find n in which b_n is smaller than the error
   d. The sum is the partial sum of the first n terms
2. **Taylor Inequality**
   If $|f^{(n+1)}(x)| \le M$ for $|x-a| \le d$, then the $R_n =$
   $|R_n(x)| \le \frac{M}{(n+1)!} |x-a|^{n+1}$ for $|x-a| \le d$.
3. **Integral Test**

$$\int_{n+1}^{\infty} f(x)\,dx \le \underbrace{s - s_n}_{R_n} \le \int_n^{\infty} f(x)$$

Common Power Series Representations

1. $\frac{1}{1-x} = 1 + x + x^2 + x^3 \ldots = \sum_{n=0}^{\infty} x^n \qquad |x| < 1$
2. $e^x =$ $\quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
   for all x
3. ln (1+x) = $\quad -1 < x <= 1$
   $\ln|1+x| = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$
4. sin x = for all x
   $x^1 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
5. cos x = for all x
   $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
6. $\tan^{-1} x =$ $\quad -1 <= x <= 1$
   $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$
7. $(1 + x)^k =$
   $1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 \ldots$
   $\sum_{n=0}^{\infty} \binom{k}{n} x^n \qquad \binom{k}{n} = \frac{k(k-1)\cdots(k-n+1)}{n!}$
   $\binom{k}{0} = 1 \qquad k = \text{any real number} \qquad |x| < 1$

Taylor Series

$$\sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{2!}(x-a)^3 + \ldots$$

Maclaurin Series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 \ldots$$

Other Stuff

a series is absolutely convergent, it also converges
alf of the interval is the Radius of Convergence
e sequence r^n is only convergent if $-1 < r <= 1$
4. Test endpoints of intervals