Math Notes – Single Variable Calculus

Derivatives
Sin u = Cos u * u'
Cos u = –Sin u * u'
Tan u = $\sec^2$u * u'
Sec u = (Sec u * Tan u)u'
Csc u = – (csc u * cot u)u'
Cot u = – ($\csc^2$u)u'
Arcsin u = u' / $(1 – u^2)^{1/2}$
Arccos u = – u' / $(1 – u^2)^{1/2}$
Arctan u = u' / 1+ u^2
Arcsec u = u' / |u| * $(u^2 – 1)^{1/2}$
Arccsc u = – u' / |u| * $(u^2 – 1)^{1/2}$
Arccot u = – u' / 1 + u^2

Integrals
Sin u = – cos u + C
Cos u = sin u + C
Tan u = – ln |cos u| + C
Sec u = ln |sec u + tan u| + C
Csc u = – ln |csc u + cot u| + C
Cot u = ln |sin u| + C
$\sec^2$u = tan u + C
$\csc^2$u = – cot u + C
Sec u * Tan u = sec u + C
Csc u * Cot u = – csc u + C
du / $(a^2 – u^2)^{1/2}$ = arcsin (u/a) + C
du / $a^2 + u^2$ = (1/a) arctan (u/a) + C
du / $u(u^2 – a^2)^{1/2}$ = (1/a) arcsec (|u|/a) + C

Power Reducing Formulas
$\sin^2$u = (1 – cos2u)/2
$\cos^2$u = (1 + cos2u)/2
$\tan^2$u = (1 – cos2u)/(1 + cos2u)

Double Angle Formulas
Sin2u = 2sinucosu
Cos2u = $\cos^2$u – $\sin^2$u = 2$\cos^2$u – 1 = 1 – 2$\sin^2$u
Tan2u = (2tan u) / (1 – $\tan^2$u)

Strange Integrals
Fnint(tan x) = ln |sec x|
Fnint(sec x) = ln |sec x + tan x|

Strange Identities
Sin A Cos B = (sin(A - B) + sin(A + B))/2
Sin A Sin B = (cos(A - B) - cos(A + B))/2
Cos A Cos B = (cos(A - B) + cos(A + B))/2
$\sin^2$x + $\cos^2$x = 1
1 + $\tan^2$x = $\sec^2$x
$\sec^2$x – 1 = $\tan^2$x
1 + $\cot^2$x = $\csc^2$x

Disc Method
V = **Pi** * FnInt (Outer Radius^2 – inner radius^2, x, a, b)
Rotating around x axis, in terms of x
Rotating around y axis, in terms of y
A and B are along axis of rotation

Shell Method
V = **2Pi** * FnInt (p(x)*h(x), x, a, b)
Rotating around x axis, in terms of y
Rotating around y axis, in terms of x
A and B are opposite axis of rotation
One side is equation and other is just x or y

Surface Area
S = **2Pi** * FnInt (r(x)* Rad(1 + [f '(x)]^2), x, a, b)
R(x) = distance between graph and the x axis

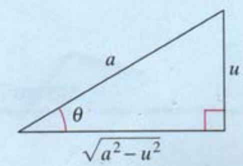
Arc Length
s = FnInt (Rad(1 + [f '(x)]^2), x, a, b)

Trigonometric Substitution ($a > 0$)

1. For integrals involving $\sqrt{a^2 - u^2}$, let

$u = a \sin\theta.$

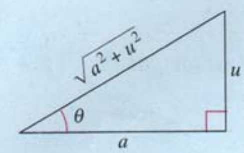
Then $\sqrt{a^2 - u^2} = a\cos\theta$, where $-\pi/2 \le \theta \le \pi/2$.



2. For integrals involving $\sqrt{a^2 + u^2}$, let

$u = a \tan\theta.$

Then $\sqrt{a^2 + u^2} = a\sec\theta$, where $-\pi/2 < \theta < \pi/2$.



3. For integrals involving $\sqrt{u^2 - a^2}$, let

$u = a \sec\theta.$

Then $\sqrt{u^2 - a^2} = \pm a\tan\theta$, where $0 \le \theta < \pi/2$ or $\pi/2 < \theta \le \pi$. Use the positive value if $u > a$ and the negative value if $u < -a$.

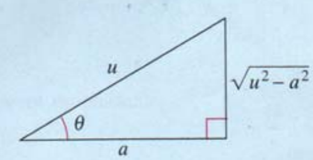


Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \ne -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int a^x\,dx = \frac{a^x}{\ln a}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
15. $\int \sinh x\,dx = \cosh x$
16. $\int \cosh x\,dx = \sinh x$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$

*19. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

*20. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}|$